Fuzzy Random Walk

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Abstract

Random walks applied to radiosity use random lines to simulate the distribution of luminous power. Noise effects in the final image are inherent in these methods. We present an improvement that reduces this noise, avoiding the sharp transitions between every patch and its coplanar neighbours, and obtaining a smoother final image with a negligible increase in computational cost.
The main idea involved in our improvement is the distribution of a part of the incoming to a patch power between its neighbours. That is, part of the power splats the neighbour patches. Our approach is based on Information Theory concepts, as uncertainty. Reductions of the mean square error to less than a half have been obtained in our tests.

Keywords: Random Walk, Radiosity, Splatting, Uncertainty

1 Introduction

Random walks are used in rendering, and particularly in radiosity, to simulate the distribution of power in a scene [1]. Random lines simulate the interreflections of light between the patches in which the scene is discretized. One of the main drawbacks of these methods is the high number of lines needed to obtain an acceptable result. Undersampling produces an undesired noise effect in the resulting images. This effect can be reduced obviously by increasing the number of paths and so the computational cost, but our interest is to reduce the noise by other means.

The fuzzy random walk introduced in this article obtains a sensitive reduction of the noise effects in the final image with a very low increase of computational cost. This improvement is based on the idea of splatting part of the power carried by a line to the neighbour patches of the receiver patch.

We borrow from Information Theory the concept of uncertainty of an event. Uncertainty just indicates how difficult is an event to happen. In our case, the event is the intersection of a patch by a random line. So, if the uncertainty of an intersection is high, a notable fraction of power will be splatted, and vice versa.

This paper is organized as follows. In next section we will briefly refer to previous work. The description of our contribution is in section 3. Section 4 presents the results, both error graphs and images. Finally, in last section we present the conclusions and future work.

2 Previous work

We have incorporated our idea to the radiosity context, and specifically to breadth-first shooting random walk [1] where each iteration simulates one bounce of the light (the first iteration deals with the primary power, and so on).

[6] presents a Monte Carlo ray tracing approach applied to non-diffuse environments in which noise effects are reduced by considering the knowledge of the neighbourhood of the paths (footprints).

Random walks applied to global illumination suppose the transmission of particles (or photons) using random lines. Each line communicates two points in the scene, and power is transmitted between these points. This communication can be seen as an information transmission and so it can be studied from Information Theory point of view [2] [4] [3]. We borrow from Information Theory the concept of uncertainty of an event. If the probability of an event k is p_k, its uncertainty is given by

$$-log_2 p_k \tag{1}$$

The uncertainty means how difficult is the event to happen. If the probability of the event is close to 1, the uncertainty is close to zero, and conversely, if the event is very unlikely, its probability is close to zero and its uncertainty is very high.

3 Fuzzy random walk

Radiosity random walks distribute the power between the patches using random lines. Each random line carries some amount of power from the patch where the line is origined to the hit patch.

The idea involved in fuzzy random walk is the distribution of part of incoming power to the neighbour patches of the hit patch (splatting), obtaining a reduction of the aliasing in the resulting image.

We have to determine the percentage of incoming power that has to be splatted. A first naive idea could be the distribution of a fixed percentage, but it is not difficult to see that there are some situations in which it seems more natural to splat part of the power to the neighbours, for instance when the area of the receiver patch is small in relation with the distance from the origin. It can be observed in Fig. 1: the distribution of power from the blue box to the patch i in the table probably needs less splatting than the distribution from the light to the pink box, since in this last case the determination of patch j

as receiver patch is more accidental than the first case (note that the patches in which is divided the pink box are smaller than the ones in the table, and moreover the distance of the intersection is clearly bigger in the second case).

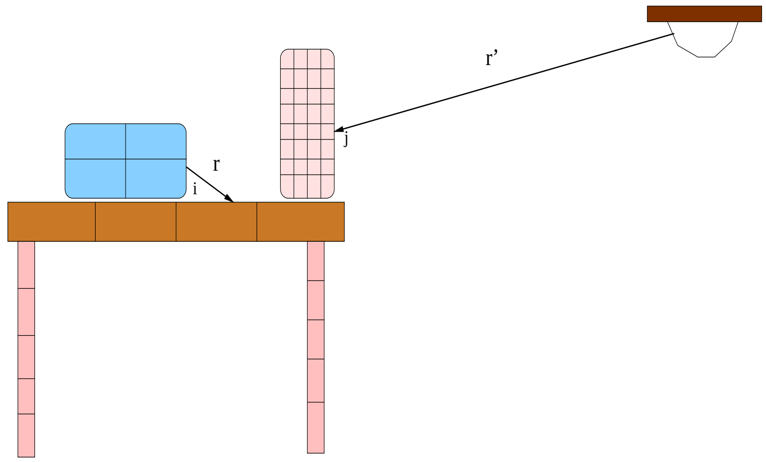


Figure 1: *The determination of patch j as receiver patch is more accidental than in the case of patch i.*

To deal with a more accurate distribution, we borrow from Information Theory the concept of uncertainty (see section 2). In the case of the radiosity random walk, we consider the probability of a line exiting from a point i to hit patch j. This is the fraction of visibility of patch j from point i (without considering any occlusion). If we consider the hemisphere of radius R (being R the distance between point i and patch j) and centered in point i, the probability can be seen as the ratio of the projection of patch j over the hemisphere and the area of the hemisphere:

$$\frac{A_j cos\theta}{2\pi R^2} \tag{2}$$

where θ is the angle between the normal of patch j and the line (note that this approximation can lead occasionally to values greater than one; in this case, we set the uncertainty to zero) (see Fig. 2).

The idea of how much accidental is a patch intersection matches completely with the concept of uncertainty. So it is natural to consider the uncertainty of every intersection (see equation (3)) in the determination of the percentage of power that has to be splatted.

$$-\log_2 \frac{A_j cos\theta}{2\pi R^2} \tag{3}$$

This value will be high if the intersection is unlikely (i.e. if the projection of patch j is small in relation with the distance of the intersection). Conversely,

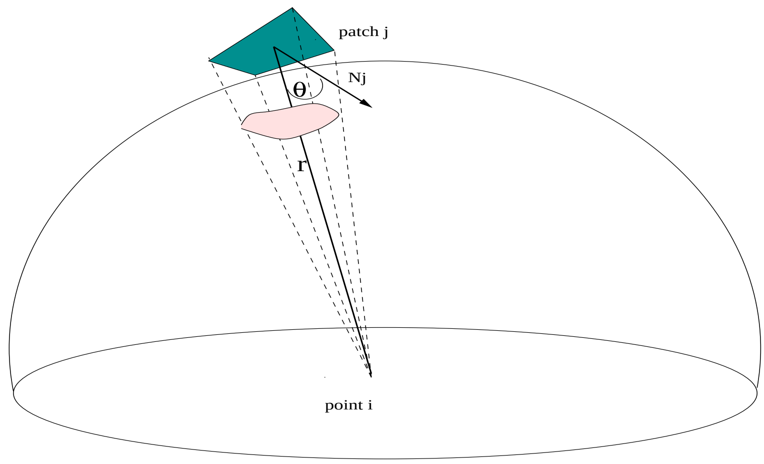


Figure 2: *Projection of patch j on the hemisphere of radius R centered in patch i*

if it is very probable to reach patch j with a random line originated in i, the uncertainty will be lower. In this last case, we consider that the distribution of power that carries out the line is more accurate than in the first case. So, the uncertainty is a good measure of how accurate the distribution of power is: the lower the uncertainty, the higher the accuracy. Then it is reasonable to establish the percentage of power that has to be splatted to the neighbour patches in function of the uncertainty. We map the uncertainty to a value between 0 and 1 that corresponds to the fraction of power that should be splatted. This is done using the following heuristic: if the uncertainty is lower than a threshold α we map to 0; if it is greater than another threshold β we map to 1 ($0 < \alpha < \beta$)); and if it is between α and β we map to the corresponding value between 0 and 1.

Note that occlusions are not taken into account in the computation of the uncertainty. This introduces an error and a bias in the final results. However, this error does not have any important repercussion, because there is a compensation in the power received for every patch, favoured by the fact that we are not using point sources.

We also have to distribute the splatted power between the neighbour patches. Fig. 3 gives us an idea of how it is done. We distribute power to the eight coplanar neighbour patches (if they exist). The amount of power that is distributed to each neighbour is given in equation (4). This corresponds to the naive idea of distributing more power to the patches that are closer to the intersection point. Note that this expression is similar to the Epanechnikov kernel used in density estimation [5].

$$w_k = \frac{1}{7} \frac{\left(\sum_{i=1}^{8} d_i^2\right) - d_k^2}{\sum_{i=1}^{8} d_i^2} \tag{4}$$

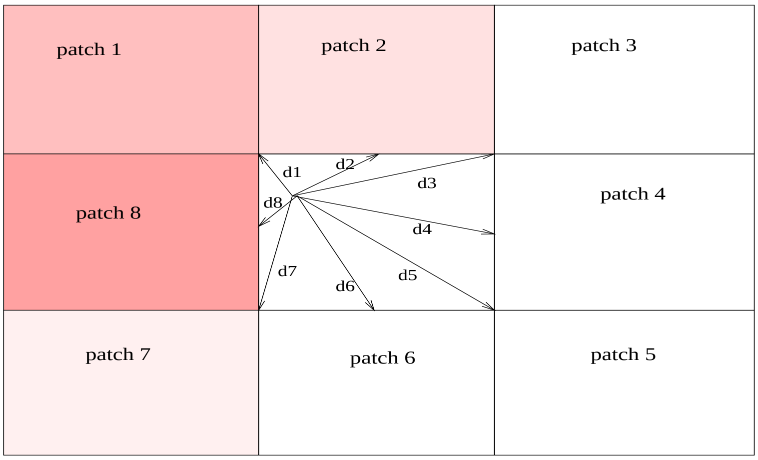


Figure 3: *The distribution of power to the neighbour patches is related to the position of the intersection point.*

4 Results

A notable reduction of the aliasing is observed with the new method, being the increase in cost nearly negligible. The application of splatting introduces a small bias that produces an unnoticeable effect on the resulting images. To avoid the bias effects in the comparisons, we have computed the error by comparing each image with its respective reference image.

Different values of the thresholds α and β (see Section 3) have been tested, given good results with $\alpha = 5$ and $\beta = 15$. We have used two different scenes[1]:

- **Scene OFFICE**

This scene represents an office with several desks, chairs and shelves. It is subdivided in approximately 7000 patches. In Fig. 6 (right) we note that incorporating fuzzy random walk the Mean Square Error (MSE) is reduced to approximately a half with a small increase of cost. We can see visual differences

[1] Images are available on ima.udg.es/$\sim$castro

in Fig. 4, where the image on the right has been obtained using fuzzy random walk with only a very small increase of execution time respect to the more noisy image on the left (without splatting).

(a) (b)

Figure 4: *Scene OFFICE. (a) Classic random walk. Number of lines: 514000. Execution time: 27 sec. (b) Fuzzy random walk. Number of lines: 511000. Execution time: 30 sec.*

- **Scene ROOM**

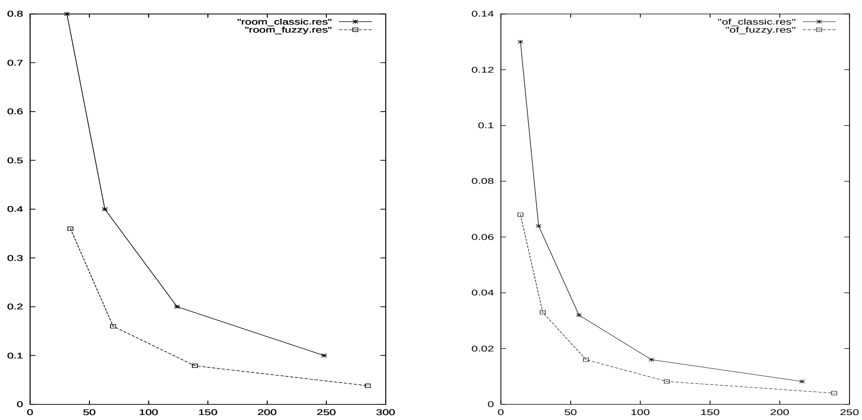
This scene represents a room with a table, some chairs and a desk. Some small objects have been placed on the table and on the desk. It has been subdivided in approximately 25000 patches. The graph in 6 (left) presents a behaviour similar to the one in the previous scene. The image in Fig. 5 shows the reduction of the aliasing (right). The increase of cost respect to the image obtained with the classic random walk (left) is small.

5 Conclusions and future work

Fuzzy random walk has obtained a notable reduction of the noise effects in the resulting images with a very small increase of computational cost. The power arriving to a patch is splatted to the neighbour patches according to the uncertainty of the hit event. This splatting is done in each iteration of the breadth-first shooting random walk algorithm. Note that the redistribution of power done in an iteration affects the unshot power for the next iteration.

(a) (b)

Figure 5: *Scene ROOM. (a) Classic random walk. Number of lines: 1433000. Execution time: 63 sec. (b) Fuzzy random walk. Number of lines: 1407000. Execution time: 70 sec.*



(a) (b)

Figure 6: *(a) Scene ROOM. Time-MSE plot. (b) Scene OFFICE. Time-MSE plot. Continous line is classic random walk, dashed line is fuzzy random walk.*

Improvements will go along the lines of changing the heuristics that determine the fraction of power that has to be splatted and how much power goes to each neighbour patch.

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