Information-Theory-Based Oracles for Hierarchical Radiosity

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Abstract. In the radiosity method scene meshing has to accurately represent illumination variations avoiding unnecessary refinements that would increase the computational cost. In this paper we present three refinement criteria for hierarchical radiosity which are based on the information content of a ray between two patches or elements and the loss of information transfer between them due to the discretisation. The results obtained with our information-theory-based oracles improve on the classic ones based on transported power, kernel smoothness, and smoothness of received radiosity.

1 Introduction

The radiosity method solves the problem of illumination in an environment with diffuse surfaces, by using a finite element approach. The scene discretisation has to accurately represent illumination variations, trying to avoid unnecessary subdivisions that would increase the computational time. A correct strategy should balance accuracy and computational cost.

In the hierarchical radiosity algorithms [1,2] the mesh is generated adaptively: when the constant radiosity assumption on a patch is not valid for the radiosity received from another patch, the refinement algorithm will refine it in a set of subpatches or elements. An oracle or refinement criterion informs us if a subdivision of the surfaces is needed, bearing in mind that its cost should remain acceptable.

In this paper, three information-theory (IT) oracles, based on entropy (information) and mutual information (information transfer), are applied to hierarchical radiosity. These oracles derive from the previous results obtained by the authors on IT applications to radiosity. The results show that our three IT oracles improve on their classic counterpart ones based, respectively, on transported power, kernel smoothness, and smoothness of received radiosity.

This paper is organized as follows: In section 2, we review different refinement oracles used in the hierarchical radiosity setting and also IT tools applied to a scene. In section 3, we present three IT oracles based on the information content of a ray and the loss of information transfer between two patches. In section 4, some experiments compare our methods with the three classic oracles. Finally, in section 5, we present our conclusions.
2 Previous Work

In this section we review three classic refinement criteria used in the hierarchical radiosity setting and also some information-theory tools applied to the visibility and radiosity of a scene.

2.1 Refinement Criteria

The radiosity method uses a finite element approach, discretising the diffuse environment into $N_p$ patches and considering the radiosities, emissivities and reflectances constant over the patches. With these assumptions, the discrete radiosity equation [3] is given by

$$ B_i = E_i + \rho_i \sum_{j=1}^{N_p} F_{ij} B_j , $$

where $B_i$, $E_i$, and $\rho_i$ are respectively the radiosity, emissivity, and reflectance of patch $i$, $B_j$ is the radiosity of patch $j$, and $F_{ij}$ is the patch-to-patch form factor, only dependent on the geometry of the scene. Form factor $F_{ij}$ is defined by

$$ F_{ij} = \frac{1}{A_i} \int_{S_i} \int_{S_j} F(x,y) dA_x dA_y $$

where $A_i$ is the area of patch $i$, $S_i$ and $S_j$ are, respectively, the surfaces of patches $i$ and $j$, $F(x,y)$ is the point-to-point form factor [4] between points $x \in S_i$ and $y \in S_j$, and $dA_x$ and $dA_y$ are, respectively, the differential areas at points $x$ and $y$.

A hierarchical refinement algorithm [1] is used to solve the system (1). The application of a good refinement criterion is fundamental for its efficiency. Many oracles have been proposed in the literature [1,5–12] and, for comparison purposes, we review here the three most-used ones

Oracle Based on Transported Power (TP). Introduced by Hanrahan et al. [1], a cheap form factor estimate $\tilde{F}_{ij}$ which ignores visibility was used to measure the accuracy of an interaction between elements $i$ and $j$. If $\max(\tilde{F}_{ij}, \tilde{F}_{ji})$ exceeds a given threshold $\epsilon$, the larger of the two elements $i$ and $j$ is subdivided using regular quadtree subdivision. Hanrahan et al. also observed that the number of subdivisions can be reduced considerably without affecting image quality by weighting $\tilde{F}_{ij}$ with the source element radiosity $B_i$ and the receiver element area $A_j$. Weighting with the receiver reflectance $\rho_i$ also further reduces the number of subdivisions without deteriorating image quality. Thus, the refinement criterion based on transported power is given by

$$ \rho_i A_i \tilde{F}_{ij} B_j < \epsilon . $$

\footnote{For a detailed review, consult Bekkaert [13].}
**Oracle Based on Kernel Smoothness (KS).** In order to improve on power-based refinement, the variation of the radiosity kernel, i.e., the point-to-point form factor $F(x,y)$, is taken into account. The refinement criterion based on kernel smoothness [6], when applied to constant approximations, is given by

$$\rho_i \max(F_{ij}^{\text{avg}} - F_{ij}^{\text{min}}, F_{ij}^{\text{max}} - F_{ij}^{\text{avg}}) A_j B_j < \epsilon,$$

where $A_j$ is the source element area, $F_{ij}^{\text{avg}} = F_{ij}/A_j$ is the average radiosity kernel value, and $F_{ij}^{\text{min}} = \min_{x \in S_i, y \in S_j} F(x,y)$ and $F_{ij}^{\text{max}} = \max_{x \in S_i, y \in S_j} F(x,y)$ are the minimum and maximum point-to-point form factors computed with pairs of random points on both elements $i$ and $j$.

**Oracle Based on Smoothness of Received Radiosity (RS).** Optimal refinement can be expected by directly estimating how well the radiosity $B_j(x)$, received at $x \in S_j$ from $S_i$, is approximated by a linear combination of the basis functions on $S_i$, i.e., by estimating the discretisation error directly. This approach was first proposed by Lischinski et al. [7]. For constant approximations, the oracle based on smoothness of received radiosity is given by

$$\rho_i \max\left( F_{ij} - \min_{x \in S_i} F_j(x), \max_{x \in S_i} F_j(x) - F_{ij} \right) B_j < \epsilon,$$

where $F_j(x) = \int_{S_j} F(x,y) dA_y$ is the point-to-patch form factor between point $x$ and element $j$.

The cheapest and most used oracle has been the TP oracle. However, it results in sub-optimal shadow boundaries and excessive refinement in smoothly illuminated areas receiving a lot of power. The KS and RS oracles were proposed as an alternative to solve this problem. However, the oracle based on kernel smoothness has the problem of unnecessary subdivisions where the kernel is unbounded, and the one based on received radiosity relies on a costly accurate computation of form factors. All in all, the additional cost invested in both smoothness-based oracles, mainly through visibility computations, may not be balanced by the obtained improvements.

### 2.2 Information-Theory Tools

In Féixas et al. [14-16], a scene was interpreted as an information channel [17] and the following definitions were introduced. *Discrete scene visibility entropy* was defined as

$$H_i^d = -\sum_{j=1}^{N_p} \frac{A_j}{A_T} \sum_{i=1}^{N_p} F_{ij} \log F_{ij},$$

where $A_T$ is the total area of the scene. The value $H_i^d$ can be interpreted as the average uncertainty that remains about the destination patch of a random walk in a scene when the source patch is known. It also expresses the information content of a random walk.
Discrete scene visibility mutual information was defined as

\[ I^d_v = \sum_{i=1}^{N_p} \sum_{j=1}^{N_p} A_j F_{ij} \log \frac{F_{ij} A_T}{A_j} \]  

and it can be interpreted as the amount of information that the destination patch conveys about the source patch, and vice versa. It is a measure of the average information transfer in a scene.

Continuous scene visibility mutual information, independent of any discretisation, expresses with maximum accuracy the information transfer in a scene and is defined as

\[ I^c_v = \int_S \int_S \frac{1}{A_T} F(x,y) \log (A_T F(x,y)) dA_x dA_y , \]  

where \( S \) is the surface of the scene. Moreover, \( I^d_v \) and \( I^c_v \) can also be considered as scene visibility complexity measures and have been generalized to the radicosity setting [14,16]. These complexity measures are interpreted as the difficulty of discretising a scene: the higher the continuous mutual information, the more difficult it is to obtain an accurate discretisation and probably more refinements are necessary to achieve a predefined precision.

The following results and proposals concerning visibility were presented in Feixas et al. [14,16]: (i) continuous mutual information \( I^c_v \) is the least upper bound to discrete mutual information \( I^d_v \), (ii) by refining the patches, \( I^d_v \) must increase (or remain the same), (iii) among different discretisations of a scene the most accurate one is the one with the highest discrete mutual information, i.e. with minimum loss of information transfer, and (iv) a global discretisation error can be given by \( \delta = I^c_v - I^d_v \) and it expresses the loss of information transfer in a scene due to the discretisation.

3 Information-Theory Oracles

In this section, three IT oracles are presented in correspondence with the three classic oracles reviewed in section 2.1. Our IT oracles are based on the visibility information content of a ray hit and the visibility discretisation error between two patches. Similar to the radicosity equation (1), where we observe that the contribution of patch \( j \) to the radicosity of patch \( i \) is given by \( p_j F_{ij} B_{ij} \), in the following refinement criteria the information or the discretisation error will be weighted by \( p_j B_{ij} \).

**Oracle Based on Transported Information (TI).** From expression (6), the term \( H^d_{ij} = -\frac{A_i}{A_T} F_{ij} \log F_{ij} \) can be considered as an element of the scene entropy matrix. Each element represents the information content of a ray hit, knowing the source patch. It is always non-negative and, in general, \( H^d_{ij} \neq H^d_{ji} \).
Similar to the TP oracle, the oracle based on transported information is given by
\[ \rho_i H_{ij}^d B_j < \epsilon , \]  
where the information \( H_{ij}^d \) has been weighted with \( \rho_i B_j \).

**Oracle Based on Information Smoothness (IS).** Similar to the KS oracle (4), we compute the minimum and maximum point-to-point form factor between two elements using area-to-area sampling, i.e., selecting pairs of random points on both elements. We take, then, the difference between the information values corresponding to minimum and maximum point-to-point form factor and the information term \( H_{ij}^d \), which is computed using the approximation
\[ F_{ij} \approx A_j \frac{1}{N_s} \sum_{k=1}^{N_s} F(x_k, y_k) \]  
where \( N_s \) is the number of lines cast between elements \( i \) and \( j \). Thus, the oracle based on the information smoothness is given by
\[ \rho_i \max(|H_{ij}^d - H_{ij}^{min}|, |H_{ij}^d - H_{ij}^{max}|) B_j < \epsilon , \]  
where \( H_{ij}^{min} = -\frac{A_i}{A_T} A_j F_{ij}^{min} \log(A_j F_{ij}^{min}) \) and \( H_{ij}^{max} = -\frac{A_i}{A_T} A_j F_{ij}^{max} \log(A_j F_{ij}^{max}) \).

**Oracle Based on Mutual Information (MI).** From definition (7), the term
\[ R_{ij}^d = \frac{A_i}{A_T} F_{ij} \log \frac{F_{ij} A_T}{A_j} \]  
can be considered as an element of the symmetric scene mutual information matrix, representing the information transfer between patches \( i \) and \( j \). We can observe that negative values appear when \( F_{ij} < \frac{A_i}{A_T} \). This situation reflects a very low interaction between the two patches involved.

The information transfer between two patches can be obtained more accurately if we calculate the continuous mutual information between them. Thus, from the continuous visibility mutual information (8), the continuous information transfer between patches \( i \) and \( j \) is given by
\[ R_{ij} = \int_{S_i} \int_{S_j} \frac{1}{A_T} F(x, y) \log(A_T F(x, y)) dA_x dA_y , \]  
As we have seen in section 2.2, a general discretisation error for a scene can be given by \( \delta = R^c - R^d \geq 0 \), but we are interested here in the contribution to this discretisation error of the patch-to-patch interaction i.e., in the discretisation error between two patches.

The computation of (12) can also be done, as in the IS oracle, with random lines joining both elements \( i \) and \( j \) [16]. For \( N_s \) lines, we have
\[ R_{ij} \approx A_i A_j \frac{1}{A_T} \frac{1}{N_s} \sum_{k=1}^{N_s} F(x_k, y_k) \log(F(x_k, y_k) A_T) , \]
where \(x_k\) and \(y_k\) are, respectively, the endpoints on patches \(i\) and \(j\) of the \(k\)-th line.

From expression (11), \(R_{ij}^d\) can be written as

\[
R_{ij}^d = \frac{A_i A_j}{A_T} \log \left( \frac{F_{ij}}{A_j} \right) \tag{14}
\]

Now, taking \(\frac{F_{ij}}{A_j} \approx \frac{1}{N_s} \sum_{k=1}^{N_s} F(x_k, y_k)\) [13], we obtain the visibility discretisation error between patches \(i\) and \(j\), \(\delta_{ij} = R_{ij}^d - R_{ij}^d\), which is approximated by

\[
\delta_{ij} \approx \frac{A_i A_j}{A_T} \left( \frac{1}{N_s} \sum_{k=1}^{N_s} F(x_k, y_k) \log F(x_k, y_k) \right) - \left( \frac{1}{N_s} \sum_{k=1}^{N_s} F(x_k, y_k) \right) \log \left( \frac{1}{N_s} \sum_{k=1}^{N_s} F(x_k, y_k) \right). \tag{15}
\]

As for non-negative numbers \(a_1, \ldots, a_n\) and \(b_1, \ldots, b_n\) the following inequality [17] holds

\[
\sum_{i=1}^{n} a_i \log \frac{a_i}{b_i} \geq \left( \sum_{i=1}^{n} a_i \right) \log \left( \frac{\sum_{i=1}^{n} a_i}{\sum_{i=1}^{n} b_i} \right), \tag{16}
\]

we obtain that \(\delta_{ij} \geq 0\). Observe that \(\delta_{ij}\) is symmetric: \(\delta_{ij} = \delta_{ji}\).

The fundamental idea in this approach is that the discretisation error gives us the loss of information transfer or the maximum potential gain of information transfer between two elements. Hence this difference can be interpreted as the benefit to be gained by refining and can be used as a decision criterion. It also represents the variation of the radiosity kernel. The oracle based on mutual information\(^2\) is given by

\[
\rho_b \delta_{ij} B_j < \epsilon. \tag{17}
\]

4 Results

We have implemented the classic and IT oracles on top of the hierarchical Monte Carlo radiosity [2] method of RenderPark [19] software. It should be noted here that our oracles can be used with any hierarchical radiosity method.

In Fig.1 and Fig.2 we present the results of comparing the classic oracles of section 2.1 with the new ones defined in section 3 for a test scene\(^3\). In Fig.1, a general view of the scene is shown, while Fig.2 displays a detail of it from a different perspective. Note that the chair on the bottom right with the back to the viewer in Fig.1 corresponds to the one on the right-hand side in Fig.2, which shows the shadow cast on the wall. The left column \((i)\) of both Fig.1 and Fig.2

\(^2\) This oracle was introduced, from a different perspective, in Feixas et al. [18].

\(^3\) Additional results can be found in ima.udg.es/~rgam/cgpm03.html.
corresponds to images obtained with the classic oracles, while the right one (ii) corresponds to the new IT-based ones.

In Fig.1a, the TP and TI oracles have been evaluated with only one line and a cheap form factor estimator. In Fig.1b, the KS and IS oracles have been computed with 10 random lines. In Fig.1c, the RS oracle has been calculated with 100 lines (10 point-to-patch form factors have been computed with 10 random lines for each one), while the MI oracle has been evaluated with only 10 random lines.

Observe in the images of Fig.1aii the finer details of the shadow cast on the wall by the chair on the right-hand side. Observe also the better-defined shadow on the chair on the left-hand side and the one cast by the desk. In Fig.2aii we can also see how the IT oracles outperform the classic ones (Fig.2a), especially in the much more defined shadow of the chair on the right. Observe also, the superior quality grid created on top of the table by the IS and MI oracles (Fig.2,(b,c),ii) as opposed to the classic ones (Fig.2,(b,c),i). Comparing our three IT oracles we conclude that TI is an effective, cheap cost oracle and that IS gives a very good quality mesh, although it is outperformed by the MI oracle with the same cost.

5 Conclusions

In this paper we have presented three new information-theory oracles for hierarchical radiosity. These oracles are based on the information content of a ray between two elements (transported information and information smoothness) and the loss of information transfer between them (mutual information). They have been compared with their counterpart classic ones: transported power, kernel smoothness, and received radiosity smoothness. The results obtained show the better behaviour of the IT oracles, confirming the usefulness of the information-theory approach to deal with the radiosity problem.

Acknowledgments

This project has been funded in part with grant numbers TIC-2001-2416-C03-01 of the of Spanish Government and 2001-SGR-00206 of the Catalán Government. All the images have been obtained with the RenderPark [19] software (www.renderpark.be).

References

Fig. 1. Comparison of classic (i) vs. IT (ii) oracles with a Gouraud shaded solution: (a.i) transported power (TP) vs. (a.ii) transmitted information (TI), (b.i) kernel smoothness (KS) vs. (b.ii) information smoothness (IS), (c.i) received radiosity (RS) vs. (c.ii) mutual information (MI). In (a), the TP and TI oracles have been evaluated with only one line and a cheap form factor estimator. In (b), the KS and IS oracles have been computed with 10 random lines. And in (c) the RS oracle has been calculated with 100 lines (10 point-to-patch form factors have been computed with 10 random lines for each one), while the MI oracle has been evaluated with only 10 random lines, obtaining 19000 patches. In each case, a total of 268500 rays are cast for the radiosity computation, obtaining approximately 19000 patches.
Fig. 2. Different perspective of the scene in Fig. 1, showing the grid obtained in the subdivision. The chair on the right-hand side, casting a shadow on the wall, corresponds to the one on the bottom right with the back to the viewer in Fig. 1.


